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Towards a Reconstruction of Archimedes' *Stomachion*

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In memoriam David Fowler

Abstract and Plan of Paper

The *Stomachion* is the least understood of Archimedes' works. This paper provides a reconstruction of its goal and structure. The nature of the evidence, including new readings from the Archimedes Palimpsest, is discussed in detail. Based on this evidence, it is argued that the *Stomachion* was a treatise of geometrical combinatorics. This new interpretation is made possible thanks to recent studies showing the existence of sophisticated combinatorial research in antiquity. The key to the new interpretation, in this case, is the observation that Archimedes might have focussed not on the possibility of creating many different figures by different arrangements of the pieces but on the way in which the same overall figure is obtained by many different arrangements of the pieces.

The plan of the paper is as follows. Section 1 introduces the *Stomachion*. Section 2 discusses the ancient testimonies and the Arabic fragment, while Section 3 translates and discusses the Greek fragment. Section 4 sums up the mathematical reconstruction offered in this paper, while Section 5 points at the possible intellectual background to the work. Appendix A contains a transcription of the Greek fragment, appendix B an English translation with redrawn diagrams, appendix C a reproduction of the digitally enhanced images of the pages of the palimpsest containing remains of the *Stomachion*.

I The Puzzle

The *Stomachion* is something of a poor relation. Take for instance Dijksterhuis' book [1987]:[1] it goes through all the works extant in the main Greek manuscript tradition, providing for each a detailed analysis. The only exception is the *Stomachion*, which Dijksterhuis consigns to the 'Miscellaneous'—together with works known through testimony or Arabic translation alone. His four pages of commentary are devoted

[1]The *Stomachion* is commented on at pp. 408–412.

more to the nature of the evidence than to the treatise itself (true, the nature of the evidence is especially complicated in this case, as we shall see below). As to the contents, Dijksterhuis expresses caution growing into frustration: "it [*scil.* the traditional name *loculus Archimedius*] may indicate that he studied the game from a mathematical point of view [...] he calls it necessary to discuss some of the properties of the so-called Stomachion [...] In the Greek fragment, however, we do not find much about this investigation". His conclusion, following upon a discussion of the Arabic fragment, is that it "can no longer be ascertained whether this result was the object aimed at or whether it played a part (and if so, what part) in the investigation as originally announced".[2] In short, Dijksterhuis—in his typical sobriety of judgment—offers us no indication of what the work, in his view, was about.

Knorr's detailed bibliography [1987] of studies of Archimedes since Dijksterhuis' original publication in 1938, contains a single entry related to the *Stomachion*, which has to do with an additional ancient testimony—not to Archimedes himself, but to the game he was studying [Rose 1956]. Only a handful of studies of the *Stomachion* have been published before or since, none of them going much beyond a summary of the interpretation in Heiberg.[3]

The *Stomachion* was such a poor relation already back in 1907, when the great Danish philologist J.L. Heiberg published in *Hermes* his article presenting the sensational find, in Istambul, of a new Archimedes manuscript, the Palimpsest [Heiberg 1907]. Heiberg devoted almost all that article to a preliminary transcription of the *Methodus*, which is of course a work of the greatest importance in the history of mathematics (and none of its text known at all prior to the discovery of the Palimpsest). Heiberg merely mentioned in passing that another text was also read in the same manuscript for the first time—the *Stomachion*. Heiberg's few comments there were dedicated more to the title of the work than to its contents [Ibidem: 240–241]. He postponed the edition of this small fragment to his major edition from 1915, and whatever scholarly interest could have been concentrated on the *Stomachion* was drowned in the wave of research into the *Methodus*.

Of course, Heiberg's neglect, as that of later scholarship, was the result of there being so little for us to study. The fragment of the *Stomachion* preserved in the Palimpsest occupies less than a single page, and contains no more than the introductory passage followed by a little over a single, small, theorem. This was the *Stomachion*'s original misfortune already in the thirteenth century: so little of it was

[2] [Dijksterhuis 1987], quotations from pp. 410 and 412, respectively.

[3] [Minonzio 2000] is mainly a commented collection of data about the Latin sources mentioning the game. The paper shows that some traditional ascriptions of the Latin sources concerning the game are untenable, but the mathematical commentary adds little that is persuasive. The translation of the Arabic fragment (made by I. Garofalo) is valuable and better than Suter's.

used by the maker of the Palimpsest. One would like to know how much longer the treatise was originally. Now, in the original Archimedes manuscript the *Stomachion* was the last in the sequence of the works (at least as far as extant parts go). The single extant page of the *Stomachion* constituted, almost certainly, the fifth page in an original quire, so that the work probably should have had at least three more pages. More can be said. We may compare this—the end of the original Archimedes book—to its beginning, where the first in the sequence of the works is *De planorum aequilibriis*. Remarkably, only the very *end* of *De planorum aequilibriis* is preserved, in a little over two pages from the beginning of a quire. The work surely began much earlier, and so it seems that the entire first quire of the original Archimedes book was discarded by the maker of the Palimpsest. Note, however, that the same maker has used at least some part of all the following quires as far as the quire containing the *Stomachion* page. There was a special decision, then, to discard the first quire.[4] Symmetrically, it appears quite possible that the maker has discarded the entire *last* quire of the Archimedes book. Both omissions, of first and final quire, are easy to understand (we shall see that, for the very same reason, the *Stomachion* is very difficult to read today). The extremities of books are the first to decay. It then follows that the *Stomachion* could have had as many as nearly twelve pages (though of course another small work could have intervened to end the original sequence, or not all of the last quire was used). This may be compared with a little under twenty six pages used by the First Book of *De sphaera et cylindro* (a long work, some 160 pages of Greek and translation in Heiberg's edition) or the eight pages used by the Second Book of *De sphaera et cylindro* (a short work, some 60 pages in Heiberg). In short, it is likely that the *Stomachion* was a respectable-sized work, of as many as some 90 pages in Heiberg, and that we have less than ten percent of it extant in Greek.

II Pieces for the Solution: Before the Palimpsest

Several sources—all later than Archimedes himself—refer to a game called the 'Stomachion' ('the Belly-Teaser': attested in Archimedes' Greek fragment and some readings of Magnus Felix Ennodius[5] and Decimus Magnus Ausonius[6]) or the 'Ostomachion' ('the Bone-Battler', other readings of Ennodius and Ausonius), perhaps even 'Suntemachion' ('the Slice-Fitter', perhaps to be read in the Arabic fragment of

[4]And actually the first quire only, since the dimensions of *De planorum aequilibriis* exactly fit the missing pages.

[5]Magnus Felix Ennodius (474–521 A.D.), *Carmina* II.133, title (Hartel).

[6]Decimus Magnus Ausonius (IV century A.D.), *Cento Nuptialis*, p. 147.39–56 (Green).

Archimedes, see below).[7] The consistency across time and space is remarkable: from at least as early as Archimedes himself, through Lucretius,[8] and down to the sixth century (date of Ennodius', latest datable testimony), Mediterranean children played a kind of tangram or 'Chinese Puzzle'. This was rigidly defined by a set of 14 pieces,[9] ideally made of ivory (Ennodius, Asmonius, Cæsius Bassus), that could be fitted to form either a square (implied by the Arabic fragment of Archimedes, stated in Lucretius and Cæsius Bassus), or alternatively—and much more prominently in our literary sources—the figures could be fitted to form many fantastic shapes so as to suit the player's imagination (Ausonius, Asmonius, Cæsius Bassus, who all repeat what must be a *topos*: the *Stomachion* as a metaphor for the way in which many prosodic combinations are possible from the same building-blocks). In the first case of forming a square, this was a game of patience and spatial intuition; in the second case of forming many fantastic figures, this was a game of creativity. This distinction would be crucial below, to our understanding of the game as studied by Archimedes.

The impression made by the ancient testimonies is that the game, in antiquity, was meant for young children. Perhaps Archimedes' treatise is the work of a young father.

Moving from the game to the treatise itself, we have two extant fragments. One is in the Archimedes Palimpsest, and will be discussed in the next section.

The other is preserved inside Arabic mathematical collections where a brief *kitāb*, 'treatise', is explicitly said (by a 17^{th} century scribe?) to be by Archimedes [Suter 1899]. The title provided is 'on the division of the *ṣtmāšyūn*[10] figure into fourteen

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[7]Heiberg and Dijksterhuis worry about the variant spelling in various sources, trying to establish the 'correct' one, but it is in the nature of such objects of popular culture to go through variant spellings and etymologies. 'Stomachion' is the spelling used in the Greek fragment by Archimedes himself and shall be used here.

[8]*De rerum natura* II.776–787, see [Rose 1956].

[9]The number 14 is implied by the Arabic fragment of Archimedes, as well as Ausonius and the late grammarians Ælius Festus Asmonius (IV century A.D.—Aftonius was thought to be his name for a long time until the manuscripts containing his works were better investigated) and Cæsius Bassus (I century A.D.). The last two can be found in *Grammatici Latini* VI, pp. 100–101 and 271–272 (Keil), respectively. These two authors refer to the game as *loculus Archimedius*, 'Archimedean box'. The traditional ascriptions to Marius Victorinus and Atilius Fortunatianus were proved to be wrong long since; scholarship after Heiberg has nonetheless insisted on sticking to the traditional names. See the discussion in [Minonzio 2000, part II].

[10]Semitic writings, of course, under-determine many of the vowels. Suter suggested to read this as Greek συντεμάχιον, 'Piece-Fitter', which Heiberg doubts, preferring to see in the Arabic title a simple rendition of στομάχιον: but notice that Classical Arabic avoided as a rule such consonant clusters as /st/ at a syllable's onset.

figures in relation to it'. The text is less a treatise and more a single proposition. It offers an explicit division of a square into 14 parts.[11] This explicit division is combined throughout with a calculation of the fraction each of the parts of the square is of the whole.[12] It is useful to note immediately the resulting fractions: 1/16, 1/48, 1/6, 1/24, 1/24, 1/12, 1/12, 1/24, 1/48, 1/24, (1/2)(1/6)+(1/2)(1/8), 1/12, 1/12, 1/12. The text explicitly asserts that the goal is to show that all parts stand in a rational ratio to the square, but one should note immediately how trivial the result is: clearly, this could not be the original goal of the discussion.

In fact, one is struck by the (i) great ease, (ii) redundancy and (iii) poor expression of the result ascribed to Archimedes. (i) As for great ease: there is hardly any geometrical argumentation at all in the treatise, and the reported values follow directly from the construction, all based on repeated applications of *Elementa* VI.1.[13] (ii) As for redundancy, it is crucial to see that there is no need to calculate explicitly the fractions of the parts so as to show that they stand in a rational ratio to the square. All parts result from the successive bisection or trisection of lines that serve as bases of parallelograms or triangles; that such bisections or trisection of lines automatically result in bisections of the respective parallelograms or triangles is the claim of *Elementa* VI.1. But once this is taken for granted, one can show in general that such a process of division is bound to result in rational ratios (each division always creates either half or a third of the original), and it would have been much more elegant simply to prove the general rationality in this direct way. (iii) As for poor expression, note that the fractions, with one exception, have a property much stronger than 'rationality': they stand to the square not merely in the ratio of a number to a number, but in the ratio of *one* to a number—they are all (with one exception, that of the single pentagon) *unit-fractions*. Had Archimedes wished to characterize the metrical property of the parts, this would have been a more informative characterization (in Greek, they would each have been a μέρος of the whole). In short, it is implausible that the goal of the proposition—if indeed one should apply to it the standards of achievement one normally associates with

[11]Note that Heiberg's figure for the Arabic proposition (bottom of [Archimedes 1910–15, 2:421]; see also p. 80 below) contains an error: the line DC is falsely joined, creating a square with 15 instead of 14 pieces. (This error is not in Suter's original publication, and of course it does clash with the explicit construction). It should be clarified immediately: the identity of the figure as a square is not certain, nor is Suter's edition in any sense final. This adds a fundamental dimension of uncertainty to any specific reconstruction of Archimedes' solution: the point will be discussed again below, following the quotation and discussion of the Greek fragments.

[12]Speaking of fractions in the context of ancient Greek mathematics is at best misleading, but for our present purposes the sloppiness will do no harm.

[13]The content of this basic Euclidean tool is that triangles and parallelograms that are under the same height are to each other as their bases.

Archimedes—was merely to show the rational ratio of the parts to the square.[14]

Then again, as soon as we assume that the proposition derives from Archimedes, we must assume that it is merely an excerpt from Archimedes' work—for the simple reason that the Greek fragment does not overlap with it at all. If so, we may easily assume that the excerptor, taking the proposition out of its original context, would also have to supply the proposition with a new goal. After all, it would no longer serve the original function it had had in its original context. We are therefore allowed to assume that the goal of the proposition preserved in the Arabic was, originally, not that of showing the rationality of the parts, but instead had a function in a more global context (this possibility, as we recall, was raised already by Dijksterhuis). What that may be, we may see from the Greek fragment itself.

III The *Stomachion* in the Palimpsest

It is the most crucial piece of evidence. We are considering a single page from the Archimedes Palimpsest—in fact, once again, a very complex and composite object. To understand the grounds for the new readings, this complexity must be explained.

We start with a piece of parchment owned by an anonymous collector and presently at the Walters Art Museum at Baltimore. This is a sorry sight. The page is torn in two at around the line where it was bent so as to form two pages—177 and 172—in a prayer-book. Its top half, or page 177—for much of the last century, the last page of that book —[15] is especially severely damaged by mold that ate into the parchment and destroyed considerable parts of it. The bottom half, or page 172, fared only a little better. All of this damage happened during the 20^{th} century, probably at the hands of the family that sold the book in 1998.

Indeed almost nothing can now be read with the naked eye. As explained in a previous publication [Netz, Saito, and Tchernetska 2001–2002], readings from the Palimpsest now crucially depend on digital image processing, provided by a team led by Roger Easton, Keith Knox and Bill Christens-Barry. In this particular part of the manuscript, however, the problem is exacerbated as a significant part of the original parchment is now lost so that no digital imaging can be of help. Here comes in yet another element in the composite object that we now study. Heiberg, it turns out, did not produce his transcription only from naked-eye inspection. As he explained himself in his publications, he commissioned a set of photographs of the manuscript. It seems that through the twentieth century hardly anyone paid

[14] The above considerations were not mentioned by Heiberg, who in fact claims quite casually of this proposition 'sine dubio ultima opusculi propositio'(!) [Archimedes 1910–15, 2:420].

[15] The manuscript used to have further pages, 178–185, lost and then replaced by a paper quire perhaps during the 16^{th} century, and then finally detached from the book at some stage of the 20^{th} century; see [Netz 2001].

any attention to Heiberg's mention of the existence of those photographs and that those who did assumed they were lost. Following the rediscovery of the Archimedes Palimpsest, these photographs were traced at the Royal Library in Copenhagen (where they are catalogued among the collection of photographs, and not inside Heiberg's Nachlass: this is probably why they were never consulted during the twentieth century). These are excellent images, excellently preserved. Taken in 1906, they are today, in many places—especially in such badly damaged areas as in parts of the *Stomachion* page—better than the manuscript itself. If only we could peer behind Heiberg's shoulder and illume an ultra-violet bulb! This we cannot do; but we can capture the photographs digitally, enlarge them and present them to the eye in ways much more effective than Heiberg could have had. The only unfortunate thing is that Heiberg did not produce a complete set of such photographs.[16] About two thirds of the text by Archimedes is covered as a whole. For the *Stomachion*, we have three of the four sides required for a complete set: 172 recto and verso, as well as 177 verso, but unfortunately not 177 recto. This however has mainly the ending of the previous treatise—the *Dimensio circuli*—and only the very beginning of the *Stomachion* itself. We thus are at the mercy of mold when it comes to the title of the treatise which, in fact, would be very important to read properly and may never be.[17]

Based on this evidence, a tentative transcription of the *Stomachion* fragment can be produced, here reported as appendix A. Here is the tentative translation of the introduction to the work, based on that tentative transcription:[18]

> As the so-called Stomachion has a variegated *theoria* of the transposition of the figures from which it is set up, I deemed it necessary: first, to set out in my investigation

[16]The photographs in the collection are numbered and appear to form a complete sequence. They were presented to the library by Heiberg himself, in 1916, immediately following upon the publication of his edition. So we probably have all the images Heiberg ever had: 65 in all (some are of a prayer-book 'opening', i.e. two half-sides of an Archimedes page, while others are of a single side of the prayer-book, i.e. a single half-side of an Archimedes page).

[17]For places where the Palimpsest is no longer legible, and for which there is no photograph from 1906, but which were read with apparent confidence by Heiberg, it is natural to take Heiberg's transcription itself as our source. However, experience elsewhere in the Palimpsest shows that even where Heiberg prints no dots beneath his characters, they may be mistaken. Hence such readings are inherently uncertain.

[18]It is quite sure that the title of the work is not simply 'ΑΡΧΙΜΗΔΟΥΣ ΣΤΟΜΑΧΙΟΝ' as read by Heiberg. The characters 'ΑΡΧΙΜΗΔΟΥΣ ΣΤΟΜΑ' are legible enough, and then the next line of title ends with 'ΟΝ', but then a symmetrical arrangement of the title requires considerably more characters than just 'ΧΙ' in the middle: is the title perhaps ΑΡΧΙΜΗΔΟΥΣ ΣΤΟΜΑ/ΧΙΟΥ ΠΡΩΤΟΝ, 'Archimedes' Stomachion, First <book>'?

of the magnitude of the whole figure each of the <figures> to which it is divided, by which <number> it is measured; and further also, which are <the> angles, taken by combinations and added together; <all of the above> said for the sake of finding out the fitting-together of the arising figures, whether the resulting sides in the figures are on a line or whether they are slightly short of that <but so as to be> unnoticed by sight. For such considerations as these are intellectually challenging;[19] and, if it is a little short of <being on a line> while being unnoticed by vision, the <figures> that are composed are not for that reason to be rejected.[20]

So then, there is not a small multitude of figures made of them, because of it being possible to rotate them (?)[21] into another place of an equal and equiangular figure, transposed to hold another position; and again also with two figures, taken together, being equal and similar to a single figure, and two figures taken together being equal and similar to two figures taken together—<then>, out of the transposition, many figures are put together. So then, we write first a small theorem pointing to this <end>.

Let us concentrate first of all on the beginning of the second paragraph. Heiberg has the following Greek [Heiberg, 1910–15, 2:416]:

Ἔστι μὲν οὖν ἐξ αὐτῶν οὐκ ὀλίγων σχημάτων | ..... ο .. διὰ τὸ ..... ν .. τοῦ εἶναι | εἰς ἕτερον τόπον τοῦ ἴσου καὶ ἰσο|γωνίου σχάματος μεταῖθεμε ... | καὶ ἑτέ ..... λαμβάνοντας.

Which he translates:

Fieri igitur potest, ut inde non paucae figurae <componantur>, quia <licet> aliam partem in alium locum figurae aequalis et aequiangulae transponere aliamque substituere.

And comments 'Quid haec sibi velint, satis obscurum est' [Ibidem, 2: 417].

Why was this passage obscure to Heiberg? He did not read the crucial word—the subject of the sentence—πλῆθος (177v. col. 1, l.1) He thus read ἔστι in the sense of 'it is possible', and derived the main thrust of the sentence as reminding us that there is a variety in the figures composable by the *Stomachion*, as each figure may be transferred to some other position. This is probably related to one further obstacle to Heiberg's interpretation.

[19] An anachronistic rendering of *philotechna*, '<worthy of> the love of the art'.

[20] The meaning of this last sentence is clear, and the Greek is more or less correct, but still it is likely that there is textual corruption here—as so often elsewhere in this text—and that the original made for smoother reading.

[21] The translation of a single, crucial word is difficult, as the word is both difficult to read and, likely, corrupt.

Heiberg has read carefully the ancient testimonies to the game and was mindful of the ancient *topos* of the *Stomachion* as a symbol of variety, displayed by the many possible fantastic figures it may compose. It appears that the mention of 'it is possible to...' in Archimedes immediately pointed Heiberg to that ancient *topos*, and so his mind was ready to interpret the passage as referring to the well-known fact that by moving pieces around, different shapes could be formed—a soldier or an elephant. And indeed this is what Archimedes talks about, moving figures around from one position to another. So much Heiberg understood.

And yet this reading is not even a genuine possibility. In short, if the text refers to there being many fantastic figures, then the mathematical comment about transposability is silly: why not just say that there are many figures because there are many ways of arranging the pieces? But this is worse: the claim of transposability, if referring to the creation of varying shapes, is in one sense misleading, in that there is a continuum of different shapes, in another vacuous, in that the only possible congruence relevant to the creation of varying shapes is self-congruence of the same piece before and after its transposition (and this is both trivial and silly).

Most important, however, the claim of self-congruence is simply not the claim made here by Archimedes. Let us review the crucial words:

... (i) it being possible to rotate them into another place of an equal and equiangular figure, transposed to hold another position; and again also (ii) with two figures, taken together, being equal and similar to a single figure, (iii) and two figures taken together being equal and similar to two figures taken together ...

The precise relationship between (i), on the one hand, and (ii) and (iii), on the other hand, is not certain, but it is beyond doubt that all three are contributing factors to the fact that there are many figures.

The second case (ii) must be a reference to the exchange of one figure with a (congruent) figure-combination. Analogously, (iii) is naturally read as referring to the case where there are two figure-combinations, each composed, differently, of different constituents, yet, taken in combination, giving rise to congruence. (Otherwise, indeed, the case simply collapses to case (i) repeated twice). And with this reading established for (ii) and (iii), case (i) as well is now provided with a clear meaning, as referring to the congruence of a figure with another figure or with itself, rotated. The text goes through the ways by which different figure-combinations can be congruent with each other.[22] Now, such an exchange of two separate congruent combinations, by necessity, does not give rise to a new overall shape. To the contrary, it must preserve overall shape. Thus the object of the discussion must be not the creation of many fantastic figures, but the possibility of many different arrangements with

[22]It is likely that Archimedes is merely suggestive in this passage, leaving out the clause that there can be more complex patterns as well with any number of constituents. This is in line with Greek treatment of generality elsewhere: it is hinted, rather than asserted (see [Netz 1999, Chapter 6]).

the same overall shape.

One must also remember Archimedes' introductions elsewhere. Many of Archimedes' works contain a brief introduction. Those introductions are characterized by strict mathematical sobriety: Archimedes concentrates fully on describing the mathematical nature of the theorems he is about to discuss. It is therefore much preferable to find a relevant mathematical meaning to the passage.

And of course: the game was never understood only in terms of a game of creativity, allowing the construction of many different figures. It was also a game of taking disparate figures and forming them into a square, which is indeed the only context by which the game is mentioned in our earliest testimony, that of Lucretius. And once this is considered, then Archimedes' point becomes immediately obvious: there are many ways of arranging the square, precisely because some combinations short of the square itself can be transposed with others.

Clearly, for such transpositions to take place, the transposed pieces must be at least equal (that is, of equal area) and equiangular.[23] The result of that is a new figure, in the sense of a new arrangement of lines within the square.[24] And notice

[23]The two conditions together are necessary but not sufficient for congruence, at least for figures with more than three sides. Archimedes shifts in the preface from this (uncorrect) characterization of congruence to the (correct) one, in terms of "equal and similar" figures (*Elementa* VI.14 entails that the two notions do not coincide; for an early occurrence of "equal and equiangular" see Aristotle, *Metaphysica* I 3, 1054b2). It is difficult to figure the reasons of such a shift (and of the original slip too; but notice that "equiangular" occurs only when Archimedes refers to one-to-one exchanges of figures, and with the particular geometrical configuration of the *Stomachion* this can be made with triangles only). To say "equal and similar" of two figures is the standard way in Greek mathematics to assert that they are congruent (cfr. *Elementa* XI.def.10 and XI.31 and 33; for the occurrences in Archimedes see [Archimedes 1910–15, 3:362]. Cfr. also *Timaeus* 55A).

[24]It might be argued that two separate divisions of a square into constituents do not form different figures at all, since in both cases the 'figure', in a sense, is a square. It is true that Greek mathematical terminology is ill-equipped to describe the nature of such different subdivisions, but calling them σχήματα is in fact natural: bear in mind, for instance, that σχῆμα could refer to a diagram ('let there be the same σχῆμα', e.g. Aristarchus, *De magnitudinibus et distantiis solis et lunae*, prop. 14), or specifically to a type of diagram made of a parallelogram with a network of parallel lines passing within it (constructed with the formulaic expression 'and let the σχῆμα be completed', e.g. *Elementa* II.7,8). σχῆμα in those two senses is fundamentally a network of lines, which two separate subdivisions certainly are. While he is not directly comparable to Archimedes, it is useful to have in mind Lucretius. He mentions the game so as to argue that: had the sea (for instance) been composed of many atoms each possessing a different color, the result would have been a variegated mosaic of colors and not a single-colored sea (which, according to Lucretius, is what we in fact see). So in the game: we see not the figure of the square alone, but the many different figures composing it, II.780–781: 'in quadrato cernimus esse dissimilis formas' (the reference however is not to the dif-

finally that the burden of the paragraph is on the word πλῆθος: there is, specifically, not a small multitude of such figures.

This then throws further light on the preceding paragraph. Archimedes points out there that two studies are required in order to complete the theory in question: first, the number by which each part is measured from the whole (i.e., for each part, the fraction it occupies of the whole is calculated); second, the angles resulting from the combinations of parts. This is obviously required so that we may decide which combinations of parts could be substituted for each.[25] Now, such a reading is conjectural at a crucial point: while the reference to measuring the angles resulting from the combination of the parts is certain, the reference to measuring the fractions of each of the parts is not. But this reading makes very simple mathematical sense; it is required by the repeated reference to 'equal and similar' later on in the introduction; its mathematical gist is nearly forced by the very meaningful characters μεγεθ (177r. col. 2, l.7), necessarily a reference to magnitude, that is to some metrical measure on the parts; and finally, this reading can then very naturally be taken to be a reference to what survives as the Arabic fragment. This, then, provides us with a clear grasp of the mathematical structure of the *Stomachion*: it is the study of the transpositions allowed on the square of the game, for which one requires both a measurement of each piece as a fraction of the square as a whole, and a study of the angles formed by the combinations of parts; with the thrust of the study being the πλῆθος of the resulting figures.

It is useful to explain in some detail how the study could have proceeded. Recall the results of the calculation arrived at by the fragment in Arabic. The parts are,

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ferent arrangements but the different constituent pieces). But, Lucretius notes a difference between colors and shapes. Colors cannot be different at different levels of description, but shapes can. So the figure, though composed of many different shapes, is also square, by Lucretius' terminology, 'on the outside', 784–785: 'nil officiunt obstantque figurae dissimiles quo quadratum minus omne sit extra'. In this special context, Lucretius needs to express the notion that a figure has an internal structure, yet 'on the outside' it is a square. Now, it would be rash to suggest that Archimedes and Lucretius represent a similar terminological regime; rather, it is to be insisted here that there is no easy way for them to proceed terminologically, in that they both want to say that an internal division of a shape represents, itself, a different σχῆμα or a combination of many *formae*.

[25]This should be qualified: the study mentioned in the first paragraph is—as is appropriate—more general than that of the second paragraph. The study of the angles which pieces make in combination is necessary not only for the study of whether two separate pieces are congruent, but is also required for judging whether a certain figure is at all composable: that is, whether certain pieces can be fitted together to make a certain shape. This gives rise to constraints on angles, in particular that the angles fitted together around a point should add up to four right angles. This perhaps may be Archimedes' point about angles taken together by combinations and added (we owe this observation to Stephen Menn).

as fractions of the whole, 1/16, 1/48, 1/6, 1/24, 1/24, 1/12, 1/12, 1/24, 1/48, 1/24, (1/2)(1/6)+(1/2)(1/8), 1/12, 1/12, 1/12. As explained above, these are all, technically speaking, unit-fractions, and the one exception—(1/2)(1/6)+(1/2)(1/8)—while not a canonical unit-fraction representation of that fraction (which would have involved no product of unit-fractions), is still using unit-fractions only.[26] Now, Greeks were used to calculating by unit-fractions (more below on the historical significance of this fact). We are not, and so it is easier for us to visualize the fractions by considering them as multiples of the common measure. This is 1/48, which provides us now with the following list of multiples: 3, 1, 8, 2, 2, 4, 4, 2, 1, 2, 7, 4, 4, 4. Remember that, for two chunks to be substitutable, a necessary condition is that they will be equal, and we may now see how the calculation of fractions is directly of help: the combination of the multiples 3 and 1 of 1/48, for instance, *may* in principle be substitutable with the multiple 4 of the same fraction; or the combination of the four multiples of 2 *may* in principle be substitutable with two multiples of 4, etc. This provides an upper bound to the number of configurations. The calculations run as follows. Consider first the exchange of single pieces. There are five 4s, four 2s, two 1s. The overall number of possible one-to-one permutations is thus 5!4!2!,[27] and this is the number of different configurations as well. Many-to-many permutations are simply repeated applications of many-to-one permutations, so that it suffices to consider the latter. It is enough to write the several decompositions of any number in the sequence above as a sum of lesser integers in the same sequence. There are few decompositions, and some of them can be ruled out as repeated applications of decompositions of even lesser integers (thus, 7=4+2+1 is made up of 7=4+3 combined with 3=2+1). The independent decompositions are 2=1+1; 3=2+1; 4=2+2 and 4=3+1; 7=4+3; 8=7+1. Each of them gives rise to 2 configurations (before and after the permutation; which ones among the 1s, the 2s or the 4s are chosen is immaterial, since permutations of them have already been taken into account). The contribution of the many-to-one permutations to the number of configurations is thus 2^6. The final result is $5!4!2!2^6=368640$.

This, of course, is only a necessary condition. A further condition is that of equiangularity. To see how this is considered, let us follow the first 'small theorem'

[26] Jens Høyrup [1990] has studied the problem of the expression of unit-fractions in mathematical texts. Among other things, he recalls the "peculiarities of the Arabic vocabulary. Unit fractions from 1/2 to 1/10 possess a particular name of their own while those with larger denominators require a full phrase [...] *unless* [they] can be composed from unit fractions with smaller denominators" [Ibidem: 297]. For this reason, preference was accorded by Arabic writers to expressions as the above, that appear unnecessarily contrived to our eyes. If so, it is likely that the form of the Arabic text is a minimal rewriting of the original sum, that would therefore have been (1/12)+(1/16).

[27] Of course, factorials are only a convenient shorthand.

in the *Stomachion.*[28]

> For let there be a right-angled parallelogram[29] ZΓ and let EZ be bisected by the <point> K and let ΓK, BE be joined from Γ, E; <it is> to be proved <that> ΓB is greater than BH.
>
> Let ΓK, BZ be produced and meet at Δ, and let ΓZ joined. Since EK is equal to KZ and ΓE, that is BZ, to ZΔ, so that ΓZ is greater than ZΔ, therefore <the> angle, too, <contained> by ZΔΓ is greater than the <angle contained> by ZΓΔ. But the <angles contained> by HBΔ, BΓZ are equal; for either is half a right <angle>. Therefore the <angle contained> by ΓHB is greater, too—for the <angle contained> by ΓHB is equal to the interior and opposite <angles contained> by HBΔ, HΔB—than the <angle contained> by HΓB, so that ΓB is greater than BH; therefore if ΓH is bisected at X, the <angle contained> by ΓXB shall be obtuse, for, since ΓX is equal to XH, and XB is common, the two are equal to the two; and the base ΓB is greater than BH: therefore the angle, too, is greater than the angle. Therefore the <angle contained> by ΓXB is obtuse, while the adjacent <angle> is acute; and the <angle contained> by ΓBH <is> half a right <angle> the parallelogram being set <as> equiangular; and the <angle contained> by BXH is acute; and again, neither are the remaining <sides? angles?> ΓBH equal and this is set up and divided by the attached part <*scilicet* by this arrangement only?>.

This is in a sense disappointing, as the proposition explicitly sets up as its goal not the relationship between angles, but that between sides—ΓB being greater than BH. However it is clear that the progression of the theorem is very informal and that it is not so much focussed on a single goal, as it is dedicated to elucidating a series of inequalities residing in a configuration, perhaps leading to the conclusion that a certain combination gives rise to a scalene triangle which therefore cannot be rotated to be arranged in more than one form (this might be the upshot of the last two steps). The central result concerns, indeed, angles—the inequality of the two

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[28]The figure (see Appendix A, p. 92) is drawn directly from the manuscript where, *pace* Heiberg, it is accurately drawn (with the single omission—a common scribal error—of the letter B; unless it is there, hidden by mold!). Heiberg was misled by his assumption that ancient diagrams should be correct in their metrical properties. Once the more schematic conventions of representation are understood, one looks for bisections at points that are not a bisection—and finds them! It is remarkable that, with the correct figure retrieved, one can find its traces in Heiberg's photograph as well.

[29]What Archimedes refers to here as a 'right-angled parallelogram' (or a rectangle) must in fact be a square. This is comparable to the way in which, in *Methodus* 14, the text speaks of a parallelogram—alone—when in fact a rectangle must be assumed. It is not clear how to account for such lapses of usage.

angles next to the point X. The same configuration can be traced in several ways in the figure of the Arabic fragment, e.g. we may find that the line AD is greater than the line AQ,[30] from which follow many angle inequalities, e.g. any of the angles FQC, EQG, ALZ, TLF are greater than the angles CGE, CEG, AZB, ZBE, DZG. The fruitfulness of this small proposition in terms of the investigation of angles is therefore apparent.[31]

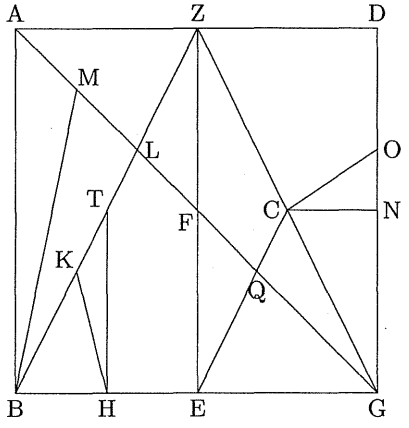

Fig. 1. The diagram of the Arabic proposition.

Apparently, the relevance of such calculations of angles to the fitting together (or its impossibility) of parts was immediately shown by Archimedes. Here is the small fragment we have of the following proposition:

> ... made (?) a right-angled, double sided ... ... , AB, having ΓA double of ΓB, having the diameter <AB>, having the thickness (?) not (?)...
> on this ?... fitting-together ?...
> ... (it is/it is not) possible to fit together along lines, with the sections having an order.
>
> And let ΓA be bisected at E, and let EZ be drawn through E parallel to BΓ. So ΓZ, ZA are squares. Let diameters be drawn ΓΔ, BE, EΔ, and let ΓH, EΔ be bisected at Θ, X, and let BΘ, XZ be joined, and let KΛ, XΞ be drawn through X, K, parallel to BΔ. Therefore through the set-out theorem, the angle at Θ of the triangle BΘΓ is obtuse, whereas it is obvious that the remainder is acute. It is obvious that ??? is greater...

Here the Greek fragment breaks off. Little survives of this final extant proposition, yet its goal is clear: by applying the result obtained above, and by introducing

[30] Superpose Arabic A on Greek B, Arabic D on Greek Γ. Extend Arabic DE in both directions to obtain Greek ΓΔ. But see below on other possible interpretations of the Arabic figure.

[31] If we take the Arabic figure and extend it so as to double it sideways, with AD=2AB, then the triangle ΓBH of the proposition above can be directly equated with the Arabic triangle ALB. This makes such a transformation on the Arabic figure very attractive; see below.

a new construction showing a detail of the geometrical configuration of the game, we can also show that for certain combinations it is (or, more likely, it isn't) 'possible to fit along lines, with the sections having an order'. We therefore get already into the actual business of checking the possibility of fitting-together (perhaps not yet that of transposition).[32]

The emerging structure of the *Stomachion* appears to be somewhat loose (though of course a broader picture would have clarified the structure). It is certainly a little surprising to see that, even though Archimedes has presented the requirements in terms of a calculation of fractions, followed by a study of angles, we start away with a study of angles and not, as implied, with the metrical calculation itself. Indeed, without a previous explicit geometrical setting up of the square of the *Stomachion*,

[32]Here an excursus on the nature of the relationship between Greek and Arabic figures—and their possible consequences for the nature of the puzzle itself—is called for. It will be noted that the Greek figures agree better, not with the Arabic figure itself, but rather with the Arabic figure stretched to form a rectangle so that AD=2AB (and then Arabic AD coincides with Greek ΓA (second Greek figure), etc.). It would obviously be preferable to have a unified interpretation of the Greek and Arabic treatments; indeed, the text of the final Greek fragment reads as if Archimedes was about to account, directly, for the congruence of pieces of the game. And so one possibility is to correct the Arabic text. In fact there is very little there to show the figure is a square. The geometrical operations assume a rectangle (the text repeatedly refers to parallelograms, and we have to assume these are rectangles since lines drawn perpendicularly to a side are taken to be parallel to the other side). Everything is based on properties of parallelism which are invariant under any stretching of the figure along either of its sides. The only indication that this is a square comes from the beginning of the text [Suter 1899: 496]: *nakhuṭṭu shaklan dhā arba'ati aḍlā'in mutawāziyatin 'alay-hi* ABGD *wa yakūnu* AD *min-hu mithla* AB, 'we draw a figure whose four sides are parallel, on which are ABGD, of which AD is equal to AB'. With inserting something such as the word *ithnāni* we can get the translation 'we draw a figure whose four sides are parallel, on which are ABGD of which AD is equal to *twice* AB', certainly not an impossible emendation, from which the identity of the Arabic and the Greek figure neatly results. (In point of fact, it can be seen that Suter's edition does not meet contemporary standards; worst of all, he consulted only two of the four manuscripts known to him: 'Da der Text gar keine Schwierigkeiten bietet, so habe ich auf eine Kollation mit den Mss. zu Oxford und London Verzichtet' [*sic*] [Suter 1899: 493]). Of course, the mention of *quadratum* in our Latin sources, as well as the desire to have an elegant puzzle, all suggest a square rather than a rectangle. However, there is nothing stopping the construction being defined for a double square, while the puzzle itself is to construct a square. But having said all the above, it should be seen that, when the Arabic figure is extended to agree with the Greek figures, it does not coincide exactly with either of them: both figures have extra lines that do not represent real divisions of the game (in the first Greek figure, this is ΓZ; in the second Greek figure, this is at least one of the mysterious lines KΛ, XΞ). Here we take the stance of referring to the divisions of the Arabic figure (possibly stretched to a double square) as the divisions of the *Stomachion*.

the strategic significance of this small proposition cannot be grasped at all. However, all of this is well within what we may expect from an Archimedean style. It has been argued in Netz (forthcoming) that Archimedes may often, deliberately, obscure his path for obtaining the main result. The first Book of *De sphaera et cylindro* is a case in point. There, having set out explicitly his main goal—measurement of the volume and surface of the sphere and its sectors—Archimedes moves on, without explanation of the strategic function of the following pieces of the sequence, to discuss various general proportion inequalities, or equalities and inequalities holding within cones and cylinders, or indeed some quite irrelevant (so it appears) relations obtaining within equilateral polygons; and then, at a stroke, he reveals to us how all those disparate results may be combined to achieve the final result.[33] It appears quite likely that the *Stomachion* had an analogous structure: following upon the explicit setting out of the goal at the beginning of the treatise (extant in Greek), Archimedes then moved on to a series of disparate geometrical observations on various geometrical configurations, showing various equalities and inequalities holding especially with angles (only the very beginning is extant in Greek). Following upon that, he then offered the explicit construction and measurement of the fractions of each of the parts (extant in Arabic). And at this point, the various disparate geometrical investigations gained a clear, coherent meaning, as they now obviously mapped onto the square of the game—allowing Archimedes to combine all his results so as to proceed to the main goal of the treatise (no longer extant in any form: see below for this strange pattern of survival).

IV Putting the Pieces Together: the *Stomachion* and Ancient Combinatorics

The simple meaning of Archimedes' words seems clear enough: the main goal of the treatise must have been to investigate what are possible ways of constructing the square.

Let us survey the possibilities for the goal of the treatise as a whole, given the above interpretation.

1. The most minimal interpretation is that Archimedes merely pointed out the possibility of certain arrangements, without claiming that they exhaust the set of all possiblities.
2. A stronger interpretation has Archimedes not only listing possible arrangements, but also claiming, even implicitly, to have surveyed them all.
3. Finally, Archimedes could also round up the discussion by explicitly calculating

[33]Such a deductive pattern makes the work of excerptors from Archimedes' treatises easy and rewarding (cfr. the existence of an Arabic *Book of Lemmas* ascribed to Archimedes).

the number of the possible arrangements as found by the study.

All three interpretations are consistent with Archimedes' words in the introduction, but with interpretation (1) one should wonder why there is any treatise at all.

The difference between interpretations (2) and (3) is more subtle and we must admit that we can not decide with any certainty between the two. We can say that of the two, interpretation (3) makes Archimedes study 'quantitative combinatorics'; while, with interpretation (2), he is engaged in a kind of 'qualitative combinatorics'—the last term sounding almost paradoxical to our ears. But perhaps this was a possible avenue for Archimedes. It may be that, for him, actually providing us with a number is less significant than simply going through a set of geometrical possibilities.[34] It is more likely in fact that Archimedes did provide an explicit count in the *Stomachion* itself. If the number of possible arrangements is finite, and it was exhaustively studied in Archimedes' treatise, then it becomes a very natural question to pose, what that number is. Archimedes' foregrounding of the word πλῆθος as, in some sense, the subject of the investigation, does suggest that the treatise aimed at some numerical value.

The treatise, in short, belongs to what we understand as combinatorics: it studies the number of possible ways by which a square may be formed out of 14 given figures.

If this is the natural reading, why was the text not read this way? Partly because it was not read very often, partly because it was not read as fully as it is presently done. But this is not the main consideration and, looking back, Heiberg's text itself seems suggestive enough of a combinatoric interpretation. The main stumbling block preventing an appreciation of the *Stomachion* as a combinatoric work lies elsewhere: until very recently, the widespread assumption was that there was no Greek combinatorics.

A series of very recent publications has dramatically changed this position so that ascribing a combinatoric treatise to Archimedes now seems like a genuine possibility. The fundamental evidence is that, in a couple of passages, Plutarch reports a calculation by Hipparchus (the great mathematician and especially astronomer of the second century B.C.), determining of the number of conjunctions Stoic logic allows with ten assertibles, without negation (103049) or with it (310954).[35] Two mathematical publications have shown that the numbers carry very precise combinatoric

[34]A close parallel can be set with the study of semi-regular solids, as shall be explained in the next section.

[35]See Plutarch, *De Stoicorum repugnantiis* 1047C–E and *Quaestiones convivales* viii 9, 732F. The manuscripts of the *Quaestiones convivales* carry the figure 101049; this is corrected from the parallel passage in the text of *De Stoicorum repugnantiis*. The second figure is given in Plutarch's manuscript as 310952; this was emended by [Habsieger *et al.* 1998].

meaning;[36] subsequently the philosophical and mathematical context in which Hipparchus' text could indeed have developed the result as a calculation of the problem implied by Chrysippus' statement has been worked out in detail [Acerbi 2003]. The existence of sophisticated Greek combinatorics is therefore no longer in question.[37]

But if Hipparchus has produced a study in combinatorics, then the most natural assumption is that he and other informed readers would have considered his work as, in part, a response to Archimedes. Of course, there could have been other combinatoric works to react to; but Archimedes would certainly be the person to react to. This may be reflected here by the curious feature of Hipparchus' calculation: his choice to offer not a single calculation, but two.

What we know of Hipparchus' work is as follows. Plutarch, in the context of his polemic against the Stoics,[38] mentions that Chrysippus' statement that there are over a million conjunctions with ten assertibles was refuted by the scientific calculation produced among others by Hipparchus, who has shown that the number was, without negation, 103049, and, with it, 310954. There is no obvious reason why Hipparchus should have presented his result in such a dual form. He could have calculated simply the number without negation (which would have been one natural way of reading Chrysippus);[39] or, if it is considered essential to allow for negation as well, we would have expected Hipparchus presenting his result as that of the second case *simpliciter*. It is true that the calculation of the second case,

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[36][Stanley 1997], followed by [Habsieger *et al.* 1998].

[37]Notice at this point that the situation with Hipparchus' numbers, and with the *Stomachion*, is neatly reversed. For Hipparchus, we have evidence for his numerical conclusions, yet no indication for his treatise itself apart from a mysterious title recorded in Arabic sources. For the *Stomachion*, we have considerable fragments of the treatise, yet no indication at all for the results arrived at. We do not attempt here a reconstruction of Archimedes' calculations. No explicit calculations are reported there, as these are surprisingly difficult to achieve, even for contemporary mathematicians, and perhaps belong to mathematics rather than to its history. Further, there is always an element of uncertainty depending on Archimedes' precise definition of what counts as a separate figure (or, indeed, of the main figure itself). At any rate, the number seem to be not more than about a myriad. Since it is natural to assume that Hipparchus, in his calculation, aimed at bigger numbers, the result seems to indicate an acceptable range. But obviously the actual combinatoric calculation ought to be pursued.

[38]The polemical tone is operative in *De Stoicorum repugnantiis*. *Quaestiones convivales* is less polemical, but it is likely that the fact was alive in Plutarch's mind as an example for Chrysippus' committing an error, not as an example of combinatoric calculation as such.

[39]In a recent notice (in [Bodnár and Netz, forthcoming]), Bodnár argues that by adding in the case with negation, Hipparchus opened himself to the potential charge, by defenders of Chrysippus, that by some other acceptable arrangements the number obtained could be bigger than a million. This underlines the fact that Hipparchus' choice to add in the second case was not self-obvious.

with negation, depends on the calculation of the first case, without it, so in this sense Hipparchus must have gone through this first calculation anyway. But this is a specious consideration: in fact, the calculation is essentially recursive and there are many other steps along the way. And yet, to later readers such as Plutarch (probably relying on earlier summaries) Hipparchus' work appeared as consisting not of one chain of calculation leading on to the number refuting Chrysippus, but as two clearly marked calculations. This is especially striking, given that Plutarch's own interest is in the higher of the numbers alone.

But then again, the dual structure of Hipparchus' treatise is easily accounted for by assuming that Archimedes' own combinatoric treatise had, itself, such a dual structure, which Hipparchus would then implicitly allude to in his own display of combinatoric skill. And indeed it is likely that Archimedes' treatise had such a dual structure: either it studies first the number of the solutions with perfect fit, followed by a study of the solutions with imperfect fit; or that (as suggested above) it first calculated an upper bound to the number of possible and then moved on to a second calculation, the exact one. In short, Archimedes has produced one calculation, providing one number; and then, making his assumptions more difficult to calculate, has extended the calculation to produce yet another, lesser number.

To sum up, the likely structure of Hipparchus' treatise allows us, first, to show the possibility of a combinatoric work by Archimedes and, second, to position the *Stomachion*, under the present reconstruction, inside a plausible historical sequence.

V Fitting the *Stomachion* in Context

The above section discusses the reaction the *Stomachion* gave rise to in at least one perceptive reader. But can we make any guesses concerning the origins of Archimedes' interest or techniques in combinatorics? In a wider perspective, can we find the intellectual context within which the *Stomachion* made sense?

It is of course not out of the question that Archimedes had some model to imitate and work against, just as for Hipparchus himself. We know too little to say anything on this issue. On the other hand, it would be simplistic to assume that Archimedes was the first author who attacked combinatorial problems — which, of course, he was perfectly capable of being. But while nothing can be said about the antecedents of the *Stomachion* in terms of past authors and works, something can be suggested concerning its conceptual antecedents. We do seem to glance, at least, something of the context which allowed Archimedes to think through a combinatoric problem.

The first context has to do with Archimedes' reliance upon unit-fractions, in the calculation of the fractions of the parts.

The possibilities of unit-fractions for combinatorics arise as follows. Unit-fractions were the preferred, perhaps the only way by which Greek authors have treated what

we call fractions.[40] Thus, for instance, what we think of as 3/4, would for the Greeks most naturally be (1/2)+(1/4)—a combination of two unit-fractions. Now, it is a central observation that any fraction can be expressed by combinations of unit-fractions, in more than one way. For instance, the same fraction above can also be represented as (1/3)+(1/4)+(1/6). There is a rich structure—not immediately apparent to surface inspection—to the decomposition of fractions by means of unit-fractions. This is a fascinating intellectual field, perhaps not studied as such but definitely well known: what we may call *merologistics*, the calculation with unit-fractions. Greek practitioners of calculation would be intimately familiar with this rich structure of equivalent expressions—which is how Archimedes would have operated with the results of the calculation of fractions extant in the Arabic fragment. In other words, the combinatoric calculation that Archimedes had effected on the square of the *Stomachion* would likely have its roots in a practical experience with a pattern of equivalence. Greek combinatorics could have been inspired, at least in one important case, by the previous experience of merologistics.[41]

Second, and perhaps most meaningful in that it has direct connection to other works in the Archimedean *corpus*, is the context of the study of semi-regular solids.

We know from a report in Pappus that Archimedes studied the class of so-called semi-regular solids—an extension of the well-known five regular solids.[42] It is also almost certain that Archimedes had a proof that there are precisely 13 such solids (given certain natural restrictions on their definition); furthermore, the problem is inherently challenging. The nature of the problem is closely comparable to that of the *Stomachion*: one had to consider in general terms some constraints on the possible combination of angles, and then, within those constraints, to enumerate the possible cases. Thus, the study of semi-regular solids likely combined geometry and calculation in more ways. In Pappus' report, the discussion is all organized around calculating the numbers of faces, sides (now usually called edges) and angles (now usually called vertices) by setting out certain relations among them and the number of vertices and sides of the polygonal faces, and it seems likely that the report represents at least some aspect of Archimedes' treatise. It may be that Archimedes used such relations to generate, from the characteristics of the vertex, the number of faces. Whatever was Archimedes' precise route, we find that the problem of semi-regular solids involved the surveying and exhausting of all possible geometrical configurations of a certain kind, deeply intertwined with techniques of the art of calculation and (possibly) indeterminate integer analysis. In other words, the mathematical structure of the *Stomachion* was not unique in the Archimedean *corpus*.

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[40] See e.g. [Knorr 1982], [Fowler 1999] and [Vitrac 1992].

[41] Details will be provided in a separate study.

[42] *Collectio mathematica* V.34–36. pp. 352–358 (Hultsch).

The mention of indeterminate integer analysis is interesting, in that it allows us to make a further connection: it immediately reminds us of another neglected treatise by Archimedes, the *Problema bovinum*. This surprising work sets out to calculate the number of the cattle of Helios, so that they satisfy seven equalities such as, e.g., that the number of white cows is equal to one-third the number of dappled Bulls and one-fourth the number of dappled cows. (Notice, again, the tight nexus of calculation and unit-fractions: all seven equalities involve unit-fractions alone).

But interest in calculation in Archimedes does not stop here: the *Arenarius*, of course, sets out to do just that—calculate. In particular, it calculates the number of grains of sand to fill the universe—a study that therefore involves question of applied geometry as well as some difficult techniques of calculation. And then some more connections can be made: Apollonius—we learn from a report in Pappus' *Collectio*, Book 2—wrote a kind of response to Archimedes' *Arenarius*, providing a technique for calculating the multiplication of the letters (taken as numerals) composing a hexameter line; and what about Eratosthenes' *Sieve*, known to us through Nicomachus' *Introductio arithmetica* I.13? This too must have used a practical calculation for the finding of all prime numbers smaller than a given number. Let us then consider together this entire family, consisting of four works by Archimedes, (i) *Stomachion*, (ii) Semi-Regular Solids, (iii) *Problema bovinum* and (iv) *Arenarius*;[43] and of three works by other Hellenistic mathematicians (all influenced by Archimedes?): (v) Eratosthenes' Sieve, (vi) Apollonius' Hexameter-counting and (vii) Hipparchus' Numbers.

Four dimensions to the family-resemblance of this group can be identified.

- All these works involve the bounding of huge and/or fuzzy objects that therefore might seem to avoid finite number. The interest is always dual: on the one hand, there is the fascination of producing a huge, seemingly unmanageable structure; on the other hand, there is the fascination of managing the structure.
- All of them could well share in a sense of variety for its own sake, a certain fascination with knowledge as play: this is of course obvious in the *Stomachion*, the *Problema bovinum* and the *Arenarius*; the pleasant variety of mathematical techniques required would have to be characteristic of the Semi-Regular Solids as well. Nothing is known of the texture of writing of Hipparchus himself; the little we see from Apollonius suggests a similar sense of the treatise as *jeu d'esprit*, and perhaps something of the same playful spirit can be glimpsed through the only element we have surviving from Eratosthenes' original treatment: namely, his use of homely metaphor for title!

[43]Archimedes refers in the *Arenarius* to a predecessor of the same work, *To (Against?) Zeuxippus*: we know nothing of this work which however, one suspects, would have belonged to the same family.

- In nearly all we can see a certain move outside the field of pure mathematics, in two directions: towards the applied mathematics of calculation, as well as towards other, non-mathematical cultural forms (that are often seen to ground the entire treatise). The role of applied calculation in all those treatises is by now obvious; the relationship to wider cultural forms is seen in the game of the *Stomachion*, probably the Platonic connection of semi-regular solids (stressed by Pappus in his report), the myth and poetry of the Cattle of Helios and the poetic trope underlying the *Arenarius*; then, obviously, the dialectic background informing Hipparchus' study, the hexameter informing that of Apollonius.
- Finally, a decisive feature shared by all these works: they were all, more or less, sidelined by later transmission. Even the one that comes closest to full survival from antiquity—the *Arenarius*—is after all preserved through a single Byzantine manuscript, the lost Codex A. In this codex, it was the only work to have its diagrams missing: the late ancient compilers of the collection of works by Archimedes, serving as model for the Byzantine Codex A, could find no copy of the *Arenarius* with the diagrams preserved: it was a rare work indeed, in late antiquity. But even so, this is the best preserved work of the works in our group. The Semi-Regular Solids as well as Apollonius' Hexameter-counting are known through reports in Pappus alone; the *Problema bovinum* survives in a manuscript tradition independent from that of Archimedes; Hipparchus' Numbers are known only through chance references by Plutarch; Eratosthenes' Sieve survives merely to the extent that Nicomachus chose to incorporate it into his general survey of arithmetic. Of course, many works in antiquity are known through such aleatory survival: but to see this pattern of loss across an entire group of works gives one pause. It appears that a genre flourished briefly—only to become obsolete and neglected not long after.

Hellenistic culture gave rise to a certain scientific genre marked by a remarkable combination: infinite ambition, playfulness, boundary-breaking. Interest in this combination did not survive later in antiquity.

Notice how puzzling the loss is—at least for the *Stomachion* itself. The game to which Archimedes refers did not become obsolete; the work itself was extant as late as the thirteenth century. Why did no one ever quote its main results? More difficult further: some late author, extant in the Arabic fragment, went to the trouble of excerpting the *Stomachion*—leaving out its main result and concentrating on the calculation of the fractions of the parts alone![44] It is true that some trace of the sense of the great variability inherent in the *Stomachion* is alive in the wider cultural

[44]Most probably this excerptor worked in Late Antiquity: it is less likely that a complete text of the *Stomachion* existed in Arabic, for then we would have been likely to hear much more of that.

tradition surrounding it. This is the gist of the *topos* recurring in Ausonius, Cæsius Bassus and Asmonius; yet these authors seem to have in mind primarily not the example of forming the square, but the example of forming many fantastic figures. In all likelihood, then, they are no longer familiar with anything like Archimedes' treatise, and so, to bring forward the idea of variability, the many forms other than the square are a more natural example than the many possible subdivisions of the square itself. This however is in a sense missing an opportunity for noting a truly striking comparison. Ausonius, for instance, is engaged in writing a cento — reworking bits and pieces out of Virgil so that fit within his own metrical lines. The sense that many different pieces of languages can be fitted together within a single prosodic template can very neatly be conveyed by an analogy not with the many fantastic figures—soldiers and elephants—but with the square itself. (It is not free verse Ausonius is producing!).

What we see, then, is that many of Archimedes' ancient readers did not bother to preserve or comment upon his combinatoric results. Among at least one of those who did read the *Stomachion*, the interest was focused purely on a single interim result, in itself representing no significant mathematical achievement.

Archimedes' work was valued—not as a piece of calculation, however, but as a piece of geometry. The proposition taken out of the *Stomachion* involved construction and measurement; the measurement was then represented in the geometrically significant terms of a rational ratio. In other words, we see that later readers had preferred geometry to calculation, so much so that they let their interest in geometry obscure some striking results of calculation available in their sources. It has been argued elsewhere that such a pattern is natural [Bodnár and Netz, forthcoming]. To briefly recapitulate the argument: Start by observing the fundamental emphasis on authoritative demonstration as the goal of a mathematical treatise. Now a calculation, however ingeniously conducted, can never display the authoritative persuasion available to qualitative geometrical arguments. There is an essential element of the empirical and piecemeal in any act of calculation. To the pedagogic schemes of later antiquity—where mathematics was the realm of Plato's models of abstract argument—the intellectual games of calculation made by Hellenistic mathematicians were of lesser interest. And so Greek readers often read, and valued, even works of calculation, through the prism of pure geometry. The curious result is that such misleading readings made by past authors have, in the case of the *Stomachion*, lead to its being misunderstood by its modern editor, as well.

Acknowledgment

In the preparation of this paper we were extraordinary fortunate in an emerging network of historians, classicists and mathematicians, who came to hear of the hypothesis regarding the *Stomachion* and, as a consequence, in many cases, came to care

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Appendix A: a Transcription of the Palimpsest Fragment of the *Stomachion*

What follows is a transcription; corrections are therefore recorded in the apparatus. Within angular brakets ⟨ ⟩ are restorations of invisible portions of the text; ( ) enclose words written as compendia. Breathings, accents and punctuation are added. We print the iota adscriptum, as in the manuscript. The figure is a faithful reproduction of the one in the manuscript.

H:416	ΑΡΧΙΜΗΔΟΥΣ ΣΤΟΜΑ	177r. col. 2
	..... ΟΝ	
	Τού λεγομένου στομαχίου ποικί-	
	λαν ἔχοντος τὰ ἐξ ὧν συνέστακε	
	σχημάτων μεταθέσεως θεωρίαν	5
	ἀναγκαῖον ἡγησάμην πρᾶτον τοῦ	
	ὅλου σ⟨χήματο⟩ς μεγέθ⟨ους θεω⟩-	

2 ante ΟΝ 6–8 literae legi nequeunt. **4** τὰ] lege τὰν; τὰν τᾶς Heiberg. **6** πρᾶτον τοῦ] fortasse πρᾶτον τοῦ τοῦ.

	ρῶν ἐκθέσθαι ⟨ε⟩ἵς τ⟨ε ἃ διαιρεῖται⟩	172v. col. 2
	ἕκαστόν τε αὐτῶν τίνι ⟨ἐστὶν μετρού⟩-	
	μενον, ἔτι δὲ καὶ ποῖαι γωνίαι συ-	
	νόδοιο λαμβανόμεναι καὶ ⟨κα⟩-	
	θῳς. εἴρηται (πρὸς) τὸ τὰς ἐναρμόσεις	5
	τῶν ἐξ αὐτῶν γεννωμένων σχα-	
	μάτων γιγνώσκεσθαι, εἴτε ἐπ' εὐ-	
H:416,10	θείας (εἰσὶν) αἱ γεννώμεναι ἐν τοῖς	
	σχάμασι πλευραί, εἴτε (καὶ) μικρῶς	
	λίπουσαι τᾶι θεωρίαι λανθά-	10
	νουσιν· τὰ γὰρ τοιαῦτα φιλότεχνα·	

καὶ ἐὰν ἐλάχιστον μὲν λίπηται, τᾶι
δὲ θεωρίαι λανθάνη, οὐ παρὰ τοῦ-
τ' ἐστιν ἔκβλητα ἃ συνίσταται.
Ἔστι μὲν οὖν ἐξ αὐτῶν οὐκ ὀλίγων σχ̣α̣μάτ(ων)

1 τ⟨ε ἃ διαιρεῖται⟩] Heiberg; nobis non legitur. **2–3** ⟨ἐστὶν μετρού⟩μενον] ἐστὶν ὁμοι̣ο̣ύ̣με̣νον Heiberg. **3–4** συνόδοιο] fortasse συνόδοις; σύνδυο Heiberg. **4–5** ⟨κα⟩θ̣ω̣ς] fortasse καθέτοι aut συνθέτοι; ⟨...⟩θάς Heiberg. **15** ὀλίγων] lege ὄλιγον.

πλῆθος, διὰ τὸ εἶλε̣ν α̣ὐ̣τὸς εἶναι̣ 177v. col. 1
εἰς ἕτερον τόπου τού ἴσου καὶ ἰσο-
γωνίου σχάματος μετατιθεμέν(ου)
καὶ ἕτεραν θέσιν λαμβάνοντος, ⟨ἔ
τι⟩ δὲ καὶ δύο σχήματα συνάμφω 5
H:416,20 ἑνὶ σχήματι ἴσων ὄντων καὶ ὁμοί-
ων τῶ ἑνὶ σχήματι ἢ καὶ δύο σχα-
H:418 μάτων συνάμφω ἴσων τε καὶ ὁμοί-
ων ὄντων δυσὶ σχήμασι συνάμφω,
πλείονα σχήματα συνίσταται ἐ- 10
κτὸς μεταθέσεως. προγραφόμε-
νον οὖν τι θεώρημα εἰς αὐτὸ συντεῖ-
νον. Ἔστω γὰρ παραλληλόγραμ-
μον ὀρθογώνιον τὸ ΖΓ καὶ δὲ διήχθω
ἡ ΕΖ τῶ Κ καὶ ἐπευζεύχθωσαν 15
ἀπὸ τῶν ΓΕ αἱ ΓΚ ΒΕ· δεικτέον μεί-
ζων (ἐστὶν) ἡ ΓΒ τῆς ΒΗ. ἐκβεβλήσ-
θωσαν αἱ ΓΚ ΒΖ καὶ συμπιπτέ-
τωσαν κατὰ τὸ Δ καὶ ἐπεζεύχθω

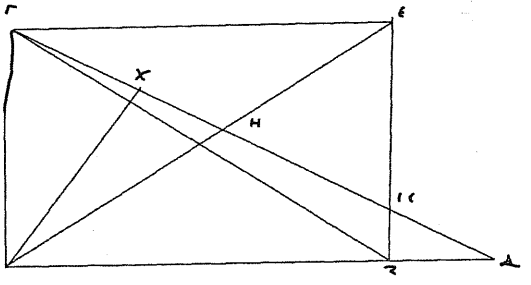
1 εἶλε̣ν α̣ὐ̣τὸς] fortasse εἰλεῖν αὐτὰ. **2** τόπου] lege τόπον. **5** σχήματα] lege σχημάτων. **7** τῶ ἑνὶ σχήματι] fortasse delendum. **10–11** ἐκτὸς] ἐκ τῆς Heiberg. **11–12** προγραφόμενον] προγράφομεν Heiberg. **14** δὲ διήχθω] requiritur δίχα τετμήσθω.

H:418,10 ἡ ΓΖ. ἐπεὶ ἴση ἐστὶν ἡ ΕΚ τῆ ΚΖ, ἴση (ἐστὶν) 172r. col. 1
καὶ ἡ ΓΕ· τουτ(έστιν) ἡ ΒΖ· τῆι ΖΔ, ὥστε
μείζων ἡ ΓΖ τῆς ΖΔ· καὶ γωνία ἄρα
ἡ ὑπὸ τῶν ΖΔΓ τῆς ὑπὸ τῶν ΖΓΔ
μείζων (ἐστὶν). ἴσαι δέ (εἰσιν) αἱ ὑπὸ ΗΒΔ ΒΓΖ· 5

ἡμίσεια γὰρ ὀρθῆς ἑκατέρα· μεί-
ζων ἄρα καὶ ἡ ὑπὸ τῶν ΓΗΒ· ἡ (γὰρ)
.. ὑπὸ ΓΗΒ ἴση (ἐστὶ) δυσὶ ταῖς ἐντὸς (καὶ)
ἀπεναντίον ταῖς ὑπὸ ΗΒΔ ΗΔΒ·
τῆς ὑπὸ τῶν ΗΓΒ· ὥστε μείζων
(ἐστὶν) ἡ ΓΒ τῆς ΒΗ. ἐὰν (ἄρα) δίχα τμη-
θῇ ἡ ΓΗ κατὰ Χ, ἔσται ἀμβλεῖα μὲ(ν)
ἡ ὑπὸ ΓΧΒ· ἐπεὶ γὰρ ἴση ἡ ΓΧ τῇ ΧΗ,
καὶ κοινὴ ἡ ΧΒ, δύο δυσὶν ἴσαι· καὶ
βάσις ἡ ΓΒ τῆς ΒΗ μείζων· καὶ

1 ἡ ΓΖ. ἐπεὶ ἴση] Heiberg; nobis non legitur. **7–9** ἡ (γὰρ) — ΗΔΒ] interpolatori tribuit Heiberg. **13–177v. col. 2,2** ἐπεὶ γὰρ — ΓΧΒ] interpolatori tribuit Heiberg.

H:418,20 ἡ γωνία ἄρα τῆς γωνίας μείζω(ν). 177v. col. 2
(ἐστὶν) ἀμβλεῖα μὲν (ἄρα) ἡ ὑπὸ ΓΧΒ· ὀξεῖα
δὲ ἡ ἐφεξῆς. ἡμίσεια δὲ ὀρθῆς ἡ
ὑπὸ ΓΒΗ, ἰσ⟨ογωνίου⟩ ὑποκειμέ-
νου τοῦ παραλληλογράμμου· ὀξεί-
α δὲ ἡ ὑπὸ ΒΧΗ καὶ ἔτι οὐδὲ ἴσαι αἱ
λοιπαὶ ΓΒΗ καὶ συνίσταται καὶ
διαιρεῖται τούτου ἔποντ⟨ι⟩ μέρει



ἔστω ⟨χ⟩ω⟨ρ⟩ίδιον διπλασιόπλευρον ⟨ὀρ⟩-
H:420 θογώνιον .πρα.σε τὸ ΑΒ διπλά 10
σιαν ἔχοντα τὴν ΓΑ τῆς ΓΒ διά
μετρον ἔχον ἔχον τὸ πάχος μὴ .λ.
.....αθ α

4 ἰσ⟨ογωνίου⟩] τοῦτο γὰρ ἐστιν Heiberg. **12** ἔχον] fortasse ἔχον τὴν

AB. **13** ante ϙθ 6–8 literae legi nequeunt. post ϙθ aliquot verba legi nequeunt.

	ποτάτ...λι.ϙρμονι...	172r. col. 2
	τ.. δύνασθαι ἁρμόζειν ὡς εὐθεί⟨(ων)⟩	
H:420,5	τῶν τομῶν ἐχουϙσῶν τάξιν. καὶ τε-	
	τμήσθω ἡ ΓΑ δίχα κατὰ τὸ Ε, καὶ	
	διὰ τοῦ Ε τῇι ΒΓ παράλληλος ἤχθω	5
	ἡ ΕΖ. (ἔστιν) οὖν τετράγωνα τὰ ΓΖ ΖϙΑ.	
	ἤχθωσαν διάμετροι αἱ ΓΔ ΒΕ ⟨Ε⟩Δ	
	καὶ τετμήσθωσαν δίχα αἱ ΓΗ ΕΔ	
	κατὰ τὰ ΘΧ καὶ ἐπεζεύχθωσαν	
H:420,10	αἱ ΒΘ ΧΖ, καὶ διὰ τῶν ⟨Χ⟩ Κ τῇ ΒΔ πα-	10
	ράλληλοι ἤχθωσαν αἱ ΚϙΑ ⟨Χ⟩Ξ. διὰ	
	τὸ προκείμενον (ἄρα) θεώρημα τοῦ	
	ΒΓΘ τριγώνου ἡ (πρὸς) τῶι Θ γωνία	
	ἀμβλεῖα, ἡ δὲ λοιπὴ ὀξεῖα ⟨φα⟩-	
	νερόν. φανερὸν δὲ (ὅτι) ⟨μ⟩εί⟨ζ(ων)⟩ ἐστὶν	15

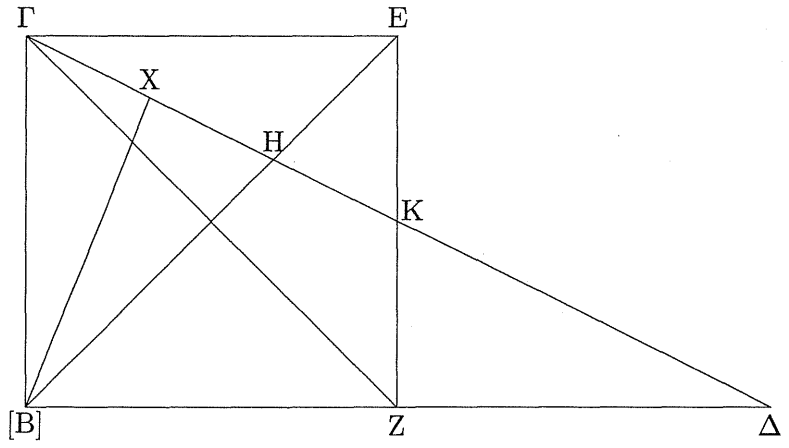
Appendix B: English Translation of the Palimpsest Fragment of the *Stomachion*

As the so-called Stomachion has a variegated *theoria* of the transposition of the figures from which it is set up, I deemed it necessary: first, to set out in my investigation of the magnitude of the whole figure each of the <figures> to which it is divided, by which <number> it is measured; and further also, which are <the> angles, taken by combinations and added together; <all of the above> said for the sake of finding out the fitting-together of the arising figures, whether the resulting sides in the figures are on a line or whether they are slightly short of that <but so as to be> unnoticed by sight. For such considerations as these are intellectually challenging; and, if it is a little short of <being on a line> while being unnoticed by vision, the <figures> that are composed are not for that reason to be rejected.

So then, there is not a small multitude of figures made of them, because of it being possible to rotate them (?) into another place of an equal and equiangular figure, transposed to hold another position; and again also with two figures, taken together, being equal and similar to a single figure, and two figures taken together being equal and similar to two figures taken together – <then>, out of the transposition, many

figures are put together. So then, we write first a small theorem pointing to this <end>.

For let there be a right-angled parallelogram ZΓ and let EZ be bisected by the <point> K and let ΓK, BE be joined from Γ, E; <it is> to be proved <that> ΓB is greater than BH.



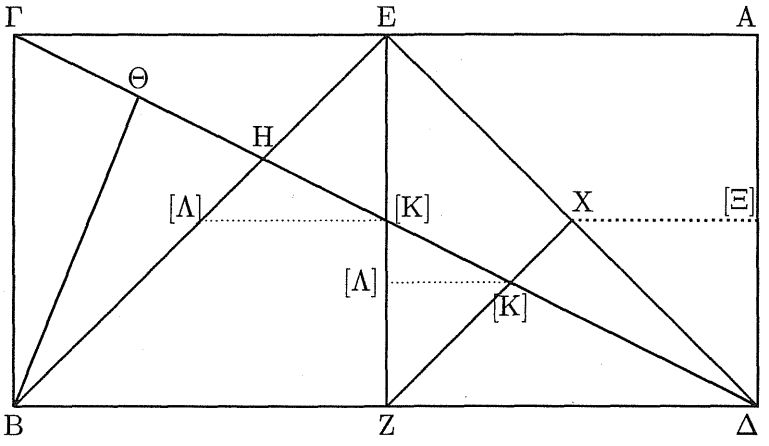
Let ΓK, BZ be produced and meet at Δ, and let ΓZ be joined. Since EK is equal to KZ and ΓE, that is BZ, to ZΔ, so that ΓZ is greater than ZΔ: therefore <the> angle, too, <contained> by ZΔΓ is greater than the <angle contained> by ZΓΔ. But the <angles contained> by HBΔ, BΓZ are equal; for either is half a right <angle>: therefore the <angle contained> by ΓHB is greater, too—for the <angle contained> by ΓHB is equal to the interior and opposite <angles contained> by HBΔ, HΔB—than the <angle contained> by HΓB: so that ΓB is greater than BH. Therefore if ΓH is bisected at X, the <angle contained> by ΓXB shall be obtuse; for, since ΓX is equal to XH, and XB is common, the two are equal to the two; and the base ΓB is greater than BH: therefore the angle, too, is greater than the angle. Therefore the <angle contained> by ΓXB is obtuse; and the adjacent <angle> is acute. And the <angle contained> by ΓBH <is> half a right <angle> the parallelogram being set <as> equiangular; and the <angle contained> by BXH is acute; and again, neither are the remaining <sides? angles?> ΓBH equal and this is set up and divided by the attached part <*scilicet* by this arrangement only?>.

... made (?) a right-angled, double sided ... ... , AB, having ΓA double of ΓB, having the diameter <AB>, having the thickness (?) not (?)...

on this ?... fitting-together ?...

...(it is/it is not) possible to fit together along lines, with the sections having an order.[45]

[45]The diagram (next page) is a reconstruction and is not contained in the palimpsest as extant. The letters in square brackets are not univocally determined by the text. Two dotted lines are alternative positions of line KΛ.



And let ΓA be bisected at E, and let EZ be drawn through E parallel to BΓ. So ΓZ, ZA are squares. Let diameters be drawn ΓΔ, BE, EΔ, and let ΓH, EΔ be bisected at Θ, X and let BΘ, XZ be joined, and let KΛ, XΞ be drawn through X, K, parallel to BΔ. Therefore through the set-out theorem the angle at Θ of the triangle BΓΘ is obtuse, whereas it is obvious that the remainder is acute. It is obvious that ??? is greater...

Appendix C: Digitally Enhanced Images of the Palimpsest

In the following two plates we reproduce the images of the palimpsest based on the photos in ordinary strobe and ultraviolet light, enhanced by digital processing. The line- and column- numbers are added in margins. The images are taken by the Rochester Institute of Technology and the Johns Hopkins University. The copyright belongs to the owner of the Archimedes Palimpsest.

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